ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE, NIGERIA

DEPARTMENT OF

MECHANICAL, AUTOMOTIVE AND PRODUCTION ENGINEERING

FIRST SEMESTER EXAMINATIONS

2017/2018 ACADEMIC SESSION

COURSE: MEE 403-Mechanical Vibrations (3 Units)

CLASS: 400 Level Mechanical & Automotive Engineering

TIME ALLOWED: 3 Hours

HOD'S SIGNATURE

INSTRUCTIONS:

a) Answer **ANY FOUR** questions
b) Make clear properly labeled sketches, graphs or diagrams where relevant or required
c) Draw fully labeled FBDs where required. See Notations at the end.
d) For calculations, you are advised to first state the steps you would use to solve the problem

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Question 1

(a) State the general ODE for analyzing the vibration of a forced damped SDOF system. What type of differential equation is it? Use Lecture notations m, k, c and f(t)

(b) Explain the steps for solving the equation.

(c) A free damped mechanical vibration system has the following elements: mass = 4 kg, k = 1 kN/m, c = 40 N-sec/m. Determine (i) damping coefficient, (ii) natural frequency of damped oscillation, (iii) logarithmic decrement and (iv) number of cycles after which the original amplitude is reduced to 20%.

Question 2

For the mechanical system shown in Figure 1, the uniform rigid bar has mass m and is pinned at O.

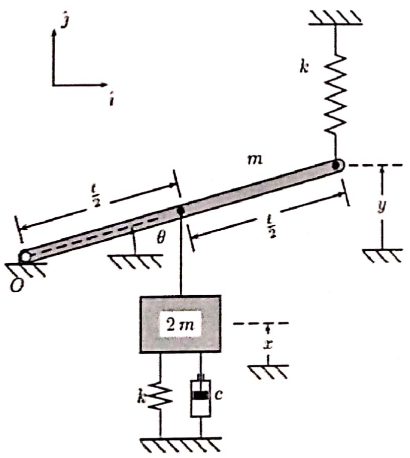


Figure 1

Assume that in the horizontal position the system is in static equilibrium and that all angles remain small. Do the following:

a) Draw the FBDs
b) Find the equations of motion
c) Determine the damping ratio and natural frequency in terms of the parameters m, c, k, and ℓ.
d) For m = 1.50 kg, ℓ = 45 cm, c = 0.125 N/(m/s), k = 250 N/m, calculate the natural frequency ω_n, the damped frequency ω_d, and damping ratio ζ. Deduce that the system is under-damped.

Question 3

For the system shown in Figure 2, the disk of mass m rolls without slip and x measures the displacement of the disk from the unstretched position of the spring.

a) Derive the equations of motion;
b) If the system is underdamped, what is the frequency of the free vibrations of this system in terms of the parameters k, c, and m;

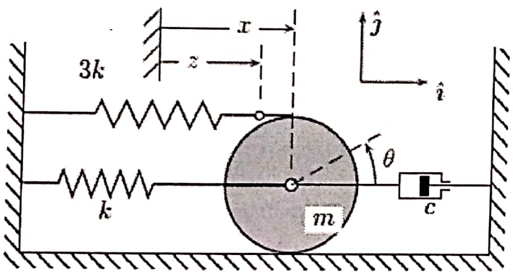


Figure 2

Question 4

(a) Explain the energy method for deriving the ODE for the case of SDOF free undamped vibration.
(b) Find the value of *k*, such that the mass-spring system described by the equation below is undergoing resonance: $8u'' + k\,u = 5\sin 6t$
(c) Solve the following initial value problem: $3u'' + 24u' + 48u = 0$, $u(0) = -5$, $u'(0) = 6$. First determine whether the system is under-, over-, or critically damped

Question 5

(a) What is a two degree of freedom (2DOF) vibrating system?
(b) Write the general form of ODE for a forced damped 2DOF system in **matrix form** and compare it with the corresponding case of SDOF system
(c) Draw the FBD for the free vibration of a 2DOF undamped spring-mass system with masses m_1, m_2, and spring constants k_1, k_2, k_3 (where k_2 is spring stiffness for the middle spring).
(d) Show that the EOM for the system in (c) can be written in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Question 6

Assume the solution for 5(d) to be of the form:

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \sin \omega t$$

Let $m_1 = m$, $m_2 = 2m$, and set $k_1 = k_2 = k_3 = k$.

(a) Derive the characteristic equation for the problem
(b) Solve for the eigenvalues, ω_i, $i = 1, 2$.
(c) Sketch the mode shapes and compare their features

Notations:

FBD = free body diagram, DOF = degree of freedom, EOM = equation of motion

Reference on Solutions of free damped SDOF systems

$u(t) = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$. undamped

$u(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$ overdamped

$u(t) = C_1 e^{rt} + C_2 t e^{rt}$ critically damped

$u(t) = C_1 e^{\lambda t} \cos \mu t + C_2 e^{\lambda t} \sin \mu t$ underdamped

Summary: the Effects of Damping on an Unforced Mass-Spring System

Consider a mass-spring system undergoing free vibration (i.e. without a forcing function) described by the equation:

$$m\,u'' + \gamma\,u' + k\,u = 0, \qquad m > 0, \qquad k > 0.$$

The behavior of the system is determined by the magnitude of the damping coefficient γ relative to m and k.

1. Undamped system (when $\gamma = 0$)

   Displacement: $u(t) = C_1 \cos\omega_0 t + C_2 \sin\omega_0 t$

   Oscillation: Yes, periodic (at natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$)

   Notes: Steady oscillation with constant amplitude $R = \sqrt{C_1^{\,2} + C_2^{\,2}}$.

2. Underdamped system (when $0 < \gamma^2 < 4mk$)

   Displacement: $u(t) = C_1 e^{\lambda t}\cos\mu t + C_2 e^{\lambda t}\sin\mu t$

   Oscillation: Yes, quasi-periodic (at quasi-frequency μ)
   Notes: Exponentially-decaying oscillation

3. Critically Damped system (when $\gamma^2 = 4mk$)

   Displacement: $u(t) = C_1 e^{rt} + C_2 t e^{rt}$

   Oscillation: No

4. Overdamped system (when $\gamma^2 > 4mk$)

   Displacement: $u(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

   Oscillation: No

Damped free vibration of SDOF system

Case		Roots	Definition
I	$c^2 > 4mk$	Distinct real roots S1, S2	Overdamping
II	$c^2 = 4mk$	Real double root	Critical damping
III	$c^2 < 4mk$	Complex conjugate roots	Underdamping

Damped free vibration of SDOF system

- Define the critical damping coefficient c_c as that value of c that makes the radical equal to zero,

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

- Define the damping factor as:

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n}$$

- Introducing the above equation into

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

- We find: $s_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$
- Then the solution can be written as:

$$x(t) = Ae^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + Be^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

Three cases of damping

- Heavy damping when c > C_c
- Critical damping c = C_c
- Light damping 0 < c < C_c

Heavy damping (c > C_c or ζ>1)

- The roots are both real. The solution to the differential equation is:

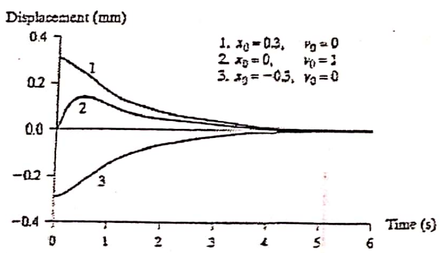
$$x(t) = Ae^{s_1 t} + Be^{s_2 t}$$

where A and B are the constants of integration. Both s_1 and s_2 will be negative because $\alpha > 0$, $\beta > 0$, and $\beta^2 = \alpha^2 - k/m < \alpha^2$. Since

$$s_1 = -\alpha + \beta, \quad s_2 = -\alpha - \beta, \text{where } \alpha = \frac{c}{2m} \quad \text{and} \quad \beta = \frac{1}{2m}\sqrt{c^2 - 4mk}$$

Thus, given any initial displacement, the mass will decay to the equilibrium position without vibratory motion. An overdamped system does not oscillate but rather returns to its rest position exponentially.

$$x(t) = Ae^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + Be^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t}$$



Critical damping (c = Cc, or ζ=1)

- Since $\beta = \frac{1}{2m}\sqrt{c^2 - 4mk}$ is zero in this case, $s_1 = s_2 = -\alpha = -c_c/2m = -\omega_n$.
- Both roots are equal and the general solution is: $x(t) = (A + Bt)e^{-\omega_n t}$.

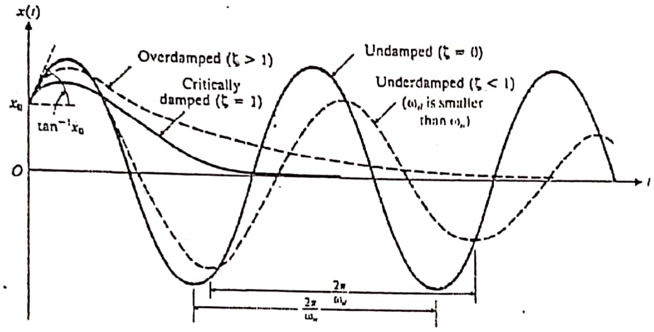
Substituting the initial conditions, $x = x_o$ at t=0 and $\dot{x} = \dot{x}_o$ at $t = 0$

$$A = x_o \text{ and } B = \dot{x}_o + \omega_n x_o$$

and the solution becomes:

$$x(t) = [x_o + (\dot{x}_o + \omega_n x_o)t]e^{-\omega_n t}$$

- The motion is again not vibratory and decays to the equilibrium position.



Light damping (0 < c < Cc or ζ<1)

- This case occurs if the damping constant c is so small that

$$c^2 < 4mk$$

- Then β is no longer real but pure imaginary.

$$\beta = i\omega^* \text{ where } \omega^* = \frac{1}{2m}\sqrt{4mk - c^2} = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

- The roots of the characteristic equation are now complex conjugate:

$$s_1 = -\alpha + i\omega^*, \qquad s_2 = -\alpha - i\omega^*$$

with

$$\alpha = \frac{c}{2m}$$

- Hence the corresponding general solution is:

$$x = e^{-\alpha t}(A\cos\omega^* t + B\sin\omega^* t) = Ce^{-\alpha t}\cos(\omega^* t - \phi_o)$$

where

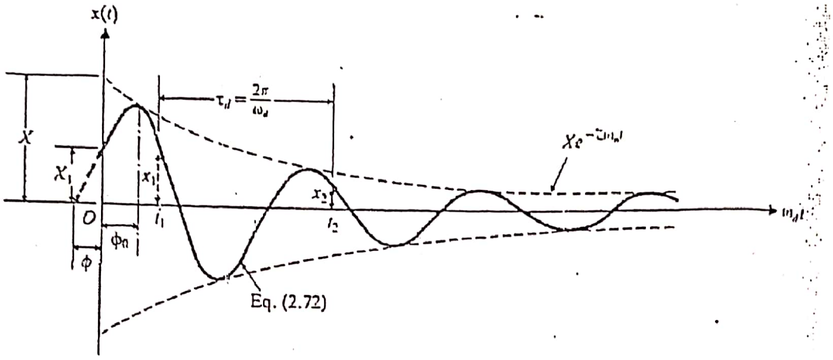
$$C^2 = A^2 + B^2 \text{ and } \tan\phi_o = B/A$$

Light damping ($0 < c < c_c$)

- The solution can also be expressed as:

$$x(t) = e^{-\zeta\omega_n t}\left(A\cos\sqrt{1-\zeta^2}\,\omega_n t + B\sin\sqrt{1-\zeta^2}\,\omega_n t\right)$$

- The roots are complex. It is easily shown, using Euler's formula that the general solution is: $x(t) = [C\cos(\omega_d t - \phi_o)]e^{-\xi\omega_n t}$
where C and ϕ are the constants of integration. The damped natural frequency ω_d is given by $\omega_d = \omega_n\sqrt{1-\xi^2}$



Light damping ($0 < c < c_c$)

- For the initial conditions

$$x(t=0) = x_o$$
$$\dot{x}(t=0) = \dot{x}_o$$

- The equation

$$x(t) = e^{-\zeta\omega_n t}\left(A\cos\sqrt{1-\zeta^2}\,\omega_n t + B\sin\sqrt{1-\zeta^2}\,\omega_n t\right)$$

can be expressed as:

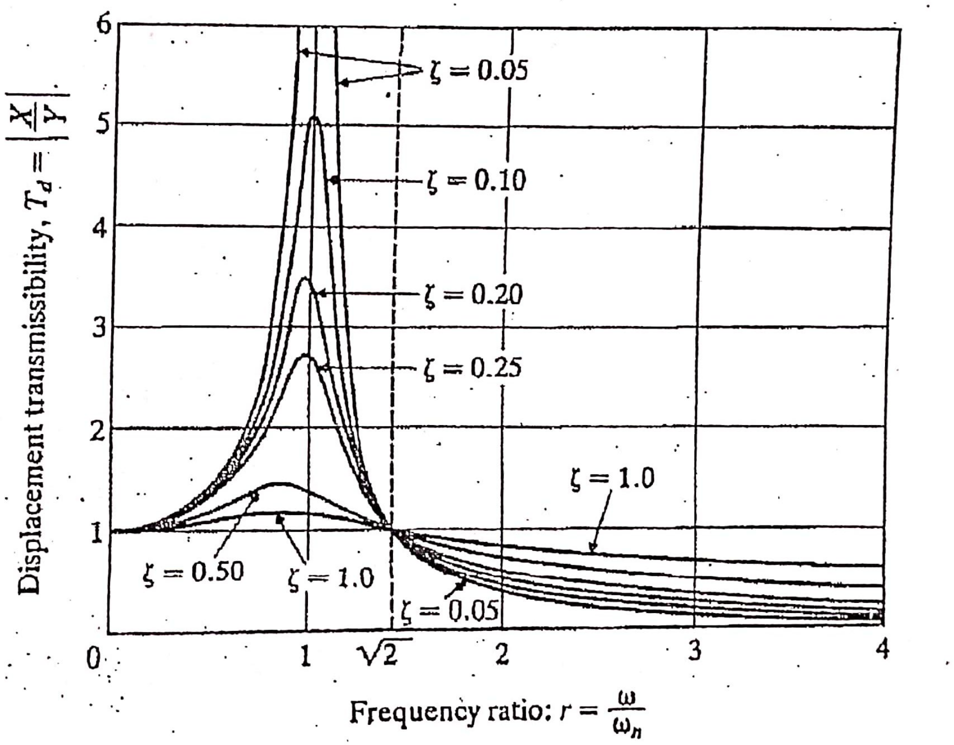
$$x(t) = e^{-\zeta\omega_n t}\left(x_o\cos\sqrt{1-\zeta^2}\,\omega_n t + \frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d}\sin\sqrt{1-\zeta^2}\,\omega_n t\right)$$

where

$$\zeta = \frac{c}{2m\omega_n}$$

Base excited systems: absolute motion

- In nondimensional form $\frac{X_o}{Y_o} = \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$
- The gain function for the absolute displacement for the base-excited system is shown in the figure.



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If the base displacement is given by a single-frequency harmonic of the form

$$y(t) = Y\sin \omega t$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 2\zeta\omega_n\omega Y\cos \omega t + \omega_n^2 Y\sin \omega t$$

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n^2 z = \omega^2 Y\sin \omega t$$

$$z(t) = Z\sin(\omega t - \phi)$$

where

- $Z = Y\Lambda(r, \zeta)$

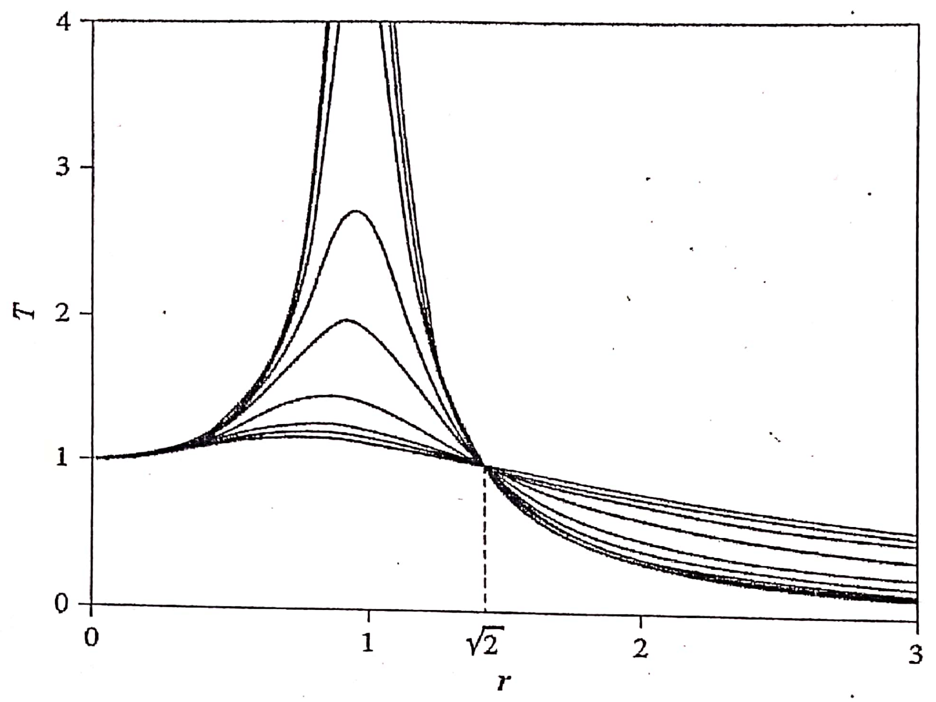
$$x(t) = X\sin(\omega t - \lambda)$$

- $\dfrac{X}{Y} = T(r, \zeta)$ $\qquad \dfrac{\omega^2 X}{\omega^2 Y} = T(r, \zeta)$

Transmissibility:

$$T(r, \zeta) = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Transmissibility ratio, T



r at max T

$$r_{max} = \frac{1}{2\zeta}\left(\sqrt{1 + 8\zeta^2} - 1\right)^{1/2}$$

$$T_{max} = 4\zeta^2\left[\frac{\sqrt{1 + 8\zeta^2}}{2 + 16\zeta^2 + (16\zeta^4 - 8\zeta^2 - 2)\sqrt{1 + 8\zeta^2}}\right]^{1/2}$$

$T(\sqrt{2}, \zeta) = 1$, independent of the value of ζ.

For $r < \sqrt{2}$, $T(r, \zeta)$ is larger for smaller values of ζ. However, for $r > \sqrt{2}$, $T(r, \zeta)$ is smaller for smaller values of ζ.

For all values of ζ, $T(r, \zeta)$ is less than one when and only when $r > \sqrt{2}$.